Year 12

Mathematics

EAS

Workbook

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nuLake Ltd
Innovative Publisher of Mathematics Texts

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Algebraic Methods 2.6

This achievement standard involves applying algebraic methods in solving problems.

Achievement	Achievement with Merit	Achievement with Excellence
• Apply algebraic methods in solving problems.	• Apply algebraic methods, using relational thinking, in solving problems.	• Apply algebraic methods, using extended abstract thinking, in solving problems.

- This achievement standard is derived from Level 7 of The New Zealand Curriculum and is related to the achievement objectives
  - manipulate rational, exponential, and logarithmic algebraic expressions
  - form and use linear and quadratic equations.
- Apply algebraic methods in solving problems involves:
  - selecting and using methods
  - demonstrating knowledge of algebraic concepts and terms
  - communicating using appropriate representations.
- Relational thinking involves one or more of:
  - selecting and carrying out a logical sequence of steps
  - connecting different concepts or representations
  - demonstrating understanding of concepts
  - forming and using a model;

  and also relating findings to a context, or communicating thinking using appropriate mathematical statements.
- Extended abstract thinking involves one or more of:
  - devising a strategy to investigate or solve a problem
  - identifying relevant concepts in context
  - developing a chain of logical reasoning, or proof
  - forming a generalisation;

  and also using correct mathematical statements, or communicating mathematical insight.
- Problems are situations that provide opportunities to apply knowledge or understanding of mathematical concepts and methods. Situations will be set in real-life or mathematical contexts.
- Methods include a selection from those related to:
  - manipulating algebraic expressions, including rational expressions
  - manipulating expressions with exponents, including fractional and negative exponents
  - determining the nature of the roots of a quadratic equation
  - solving exponential equations (which may include manipulating logarithms)
  - forming and solving linear and quadratic equations.

Factorising Complex Quadratics

When we are asked to factorise a quadratic in the form $ax^2 + bx + c$ where the 'a' term is not 1 then we need to approach the problem differently.

To factorise $5x^2 - 14x - 3$

1. Multiply the first and last terms together.

$$(5x^2 \times {}^-3) = {}^-15x^2$$

2. Find two numbers that multiply to give $^-15x^2$ and add to give the middle term of ^-14x.
3. The numbers are ^-15x and x.
4. Rewrite the quadratic, using these two values as the middle term.

$$5x^2 - 14x - 3 = 5x^2 - 15x + x - 3$$

Factorise each pair of terms using the grouping technique.

$$\begin{aligned} 5x^2 - 14x - 3 &= 5x^2 - 15x + x - 3 \\ &= 5x(x - 3) + 1(x - 3) \end{aligned}$$

6. Taking out $(x - 3)$ as the common factor.

$$= (x - 3)(5x + 1)$$

Extra Notes

First check if there is a common factor

If $a \neq 1$ first check to see whether there is a common factor that can be removed.

Consider $3x^2 + 15x + 18$

Taking out the common factor of 3

$$3x^2 + 15x + 18 = 3(x^2 + 5x + 6)$$

Then factorising bracketed terms gives

$$3x^2 + 15x + 18 = 3(x + 2)(x + 3)$$

If there is no common factor use the technique on the left.

Example

Factorise the quadratic $2x^2 - 4x - 126$

We recognise that there is a common factor for $2x^2$, ^-4x and $^-126$. So we start by taking out this common factor of 2.

$$2x^2 - 4x - 126 = 2(x^2 - 2x - 63)$$

We then factorise the bracketed term $x^2 - 2x - 63$ to get

$$\begin{aligned} 2x^2 - 4x - 126 &= 2(x^2 - 2x - 63) \\ &= 2(x - 9)(x + 7) \end{aligned}$$

Example

Factorise the quadratic $3x^2 - 11x + 6$

There is no common factor so we begin by finding two terms that multiply to give $3x^2 \times 6 = 18x^2$ and add to give ^-11x.

The terms are ^-9x and ^-2x.

Rewriting the quadratic with these new terms

$$3x^2 - 11x + 6 = 3x^2 - 9x - 2x + 6$$

Grouping $= (3x^2 - 9x) - (2x - 6)$

Factorising $= 3x(x - 3) - 2(x - 3)$

$= (x - 3)(3x - 2)$

Note: The sign changes as there is a minus outside the brackets.

Achievement – Factorise the following.

86. $2x^2 - 6x - 8$

87. $3x^2 - 12x + 9$

88. $3x^2 + 27x - 30$

89. $4b^2 + 13b + 10$

90. $3x^2 - 7x - 6$

91. $6n^2 - 23n + 20$

92. $3x^2 - 9x - 120$

93. $4x^2 + 20x - 56$

94. $2n^2 + 26n + 24$

Merit – Solve the following.

213. A company pays \$28 000 for a new forklift which the company can depreciate at the rate of 13% per annum.

The formula $V_n = A\left(1 - \frac{r}{100}\right)^n$ can be used to represent depreciation, where A is the initial cost, r the rate of depreciation, n the number of years of depreciation and V_n the value of the article after n years.

a) Find the forklift's value after 5 years.

b) Form an equation and solve it to find out how long it takes (in years) before the forklift is worth only \$3000.

214. Auckland house prices have been inflating at 14.5% pa. A house is originally purchased for \$825 000.

The formula $V = U\left(1 + \frac{r}{100}\right)^T$ can be used to represent inflation, where U is the initial cost, r the rate of inflation, T the number of years of inflation and V the value of the house after T years.

a) What is the expected price in ten years?

b) How long would you have to own the \$825 000 house for it to be worth \$2 000 000?

Excellence – Form an equation and solve it to answer the following problems.

215. Penny has been left \$125 000 from her Aunt and instead of spending it she hopes to invest it at 4.5% pa and grow it until she has a million dollars.

The formula $V = U\left(1 + \frac{r}{100}\right)^T$ can be used to represent the compound interest, where U is the initial amount, r the rate of interest, T the number of years and V the value after T years. The interest is always credited at the end of the investment year. Penny made her investment during 2000. In what year will Penny's investment be worth one million dollars?

216. Auckland house prices have been inflating at 14.5% pa while Wellington prices have only been inflating at 8.4% pa. In 2016, a house is purchased in Auckland for \$825 000 and one in Wellington for \$1.2 million.

The formula $V = U\left(1 + \frac{r}{100}\right)^T$ can be used to represent inflation, where U is the initial cost, r the rate of inflation, T the number of years and V the value of the house after T years. In what year would the house in Auckland be worth more than the house in Wellington?

Quadratic Equations

Quadratic Equations

If we are able to factorise a quadratic equation then finding the solutions is straightforward.

If we have a product of two factors that equal 0 then one (or both) of the factors must equal 0.

$$A.B = 0$$

then either $A = 0$ or $B = 0$

Therefore for a quadratic equation

$$(x+3)(x+2) = 0$$

then either $(x+2) = 0$

$$x = {}^-2$$

or $(x+3) = 0$

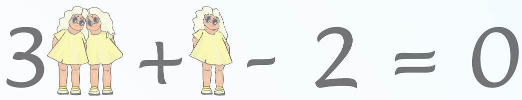
$$x = {}^-3$$

The solution is written as

$$x = {}^-3, {}^-2$$

If the quadratic is not already factorised, then we must factorise it for this method to work.

Be on the look out for quadratic equations disguised as something else.



A quadratic equation is a polynomial where the highest power of the unknown is two. A example is

$$3x^2 + x - 2 = 0$$

$$(3x-2)(x+1) = 0$$

$$x = {}^-1, \frac{2}{3}$$

but if we divide all terms by x and manipulate it the equation at first glance does not appear as a quadratic

$$3x + 1 = \frac{2}{x}$$

Yet this is an identical equation. Two other equations that can be solved as quadratic equations are

$$3x^4 + x^2 - 2 = 0$$

and

$$3x + \sqrt{x} = 2$$

In the first we substitute $z = x^2$ to get our quadratic and in the second we substitute $z = \sqrt{x}$.

Example

Solve the equation

$$2x^2 - x = 36$$

$$2x^2 - x = 36$$

Rearrange so that the equation equals zero first

$$2x^2 - x - 36 = 0$$

Factorising

$$(2x-9)(x+4) = 0$$

Setting each factor equal to 0 gives

$$(2x-9) = 0$$

$$x = 4.5$$

and $(x+4) = 0$

$$x = {}^-4$$

$$x = {}^-4, 4.5$$

Example

Solve the equation

$$3x + \sqrt{x} = 2$$

Substitute $z = \sqrt{x}$

$$3z^2 + z = 2$$

Rearrange so that the equation equals zero first

$$3z^2 + z - 2 = 0$$

Factorising

$$(3z-2)(z+1) = 0$$

Setting each factor equal to 0 gives

$$z = {}^-1, \frac{2}{3}$$

$$x = z^2$$

This would appear to give answers

$$x = 1, \frac{4}{9}$$

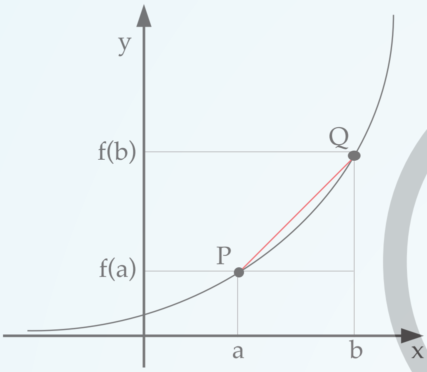
but substitution into the original equation we find the only answer is $x = \frac{4}{9}$

This is because the square root sign represents the positive root only (i.e. 1) which does not solve the original equation. We should always confirm these disguised questions by substituting back into the original equation.

The Derivative from First Principles

The Derivative

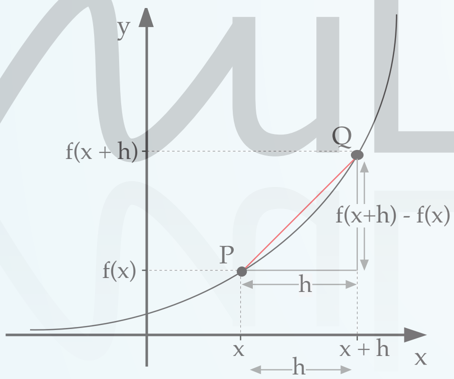
We have already established that the average rate of change of a graph is given by finding the gradient of a chord.



To calculate the gradient or average rate of the chord PQ we use the formula

$$m = \frac{f(b) - f(a)}{b - a}$$

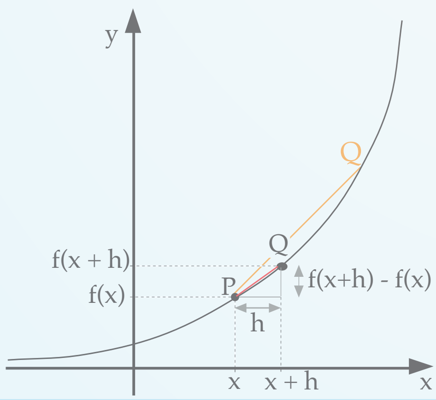
If we let the gap between a and b be h and we change our notation so P is now (x, f(x)) then Q becomes (x + h, f(x + h))



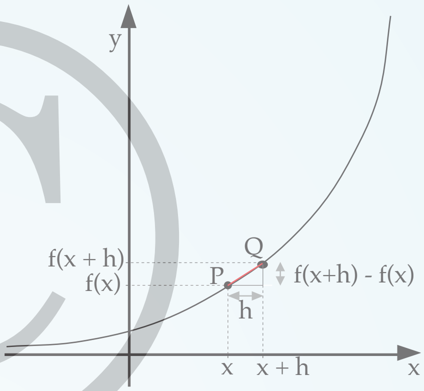
The gradient of the chord is now

$$m = \frac{f(x+h) - f(x)}{(x+h) - x}$$

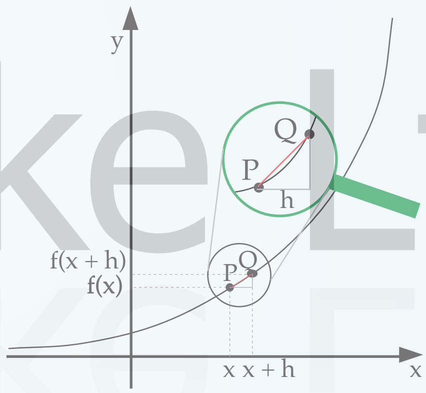
Now if we reduce the value of h, then Q gets closer to P.



If we continue to reduce the value of h then the gradient of the chord PQ gets closer and closer to the gradient of a tangent at the point P.



If the interval over which we are finding the average rate of change is made smaller and smaller, then the



limiting value of the chord's gradient as h approaches zero $[h \to 0]$ is the gradient of the tangent and the gradient of the curve at this point.

This limiting value of the gradient at a point becomes

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

This derived function f′(x), is known as the **gradient function** and the process is called **differentiation**.

When we use the formula above to calculate the derived function we are **differentiating by first principles**.

The notation f′(x) is used to denote the gradient function.

Example

Differentiate the following using the correct notation

a) $y = 4x^3 + 5x^2 - x + 2$

find $\frac{dy}{dx}$.

b) $f(x) = 2x^7 - 5x^5 + 7$

find $f''(x)$.

c) $y = 2x^2 - 7x - 15$

find the derivative of y.

If the problem starts f(x) etc. we use function notation.
If it starts with y = we use Leibniz notation.

a) $\frac{dy}{dx} = 12x^2 + 10x - 1$

differentiating once

b) $f'(x) = 14x^6 - 25x^4$

differentiating once

$f''(x) = 84x^5 - 100x^3$

differentiating again

c) Using Leibniz notation

$\frac{dy}{dx} = 4x - 7$ differentiating once

Note: We acknowledge that only first derivatives are in the syllabus, but the authors expect a number of teachers will teach the use of second derivatives in solving maximum and minimum problems.

Achievement / Merit – Differentiate using the appropriate notation.

61. $f(x) = 9x^3 + 2x + 3$ find $f'(x)$

62. $f(x) = 3x^7 - 5x^3$ find $f''(x)$

63. $y = 5x - 3x^5$ find $\frac{dy}{dx}$

64. $y = x^2 - 4$ find $\frac{dy}{dx}$

65. $f(x) = 3x^5 - 15x$ find $f''(x)$

66. $y = 4x^3 - 6x^2 + 7x + 2$ find $\frac{d^2y}{dx^2}$

67 $y = \frac{1}{2}x^2 - \frac{3}{4}x^3$ find $\frac{dy}{dx}$

68. $y = {}^-1.2x^5 + 2.5x^2 + 2$ find $\frac{dy}{dx}$

69. $f(x) = \frac{3}{4}x^3 - \frac{2}{3}x^2 + 4x - 1$ find $f''(x)$

70. $y = 16x^4 - 24x^2 + 9$ find $\frac{d^2y}{dx^2}$

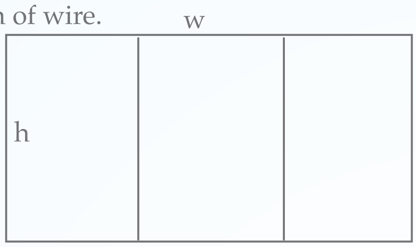
71. $f(x) = \frac{{}^-4}{5}x^4 + \frac{3}{5}x^3 + \frac{4}{3}x^2 - 2x + 5$ find $f''(x)$

72. $y = \frac{3x^2}{2} - \frac{4x^3}{9} - \frac{x}{2} - 7$ find $\frac{d^2y}{dx^2}$

Example

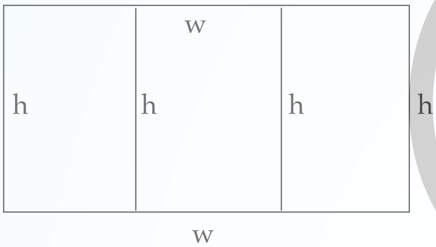
A wire frame is to be constructed out of 1200 mm of wire.

Find the height and width of the frame so that the total area is as large as possible.



Using **DELS DEBT**

Draw a diagram.



Express the problem that needs to be maximised or minimised (often in terms of two variables).

$$\text{Area} = \text{height} \times \text{whole width}$$
$$= h \times w$$
$$= hw$$

Link the two variables above with an equation.

$$1200 = 4h + 2w$$

Substitute for one variable using the link above.

$$h = \left(\frac{1200 - 2w}{4}\right)$$
$$= 300 - 0.5w$$

$$\text{Area} = w(300 - 0.5w)$$
$$= 300w - 0.5w^2$$

Differentiate.

$$\text{Area}' = 300 - w$$

Equate to zero so that you can find the turning points.

$$300 - w = 0 \qquad \text{For Max / Min}$$
$$w = 300$$

Back substitute into the original problem to find the maximum or minimum value.

$$\text{Area} = 300 \times 300 - 0.5(300)^2$$
$$= 45\,000 \text{ mm}^2$$

Test your answer by re-reading the question. No. The original question called for the height h and width w, so the correct answer is

$$w = 300 \text{ mm}$$

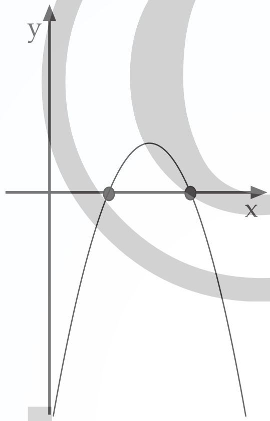
and $$h = 300 - 0.5 \times 300$$
$$= 150 \text{ mm}$$

Extra Examples

156. Find the function whose derivative is

$$\frac{dy}{dx} = 3 - x$$

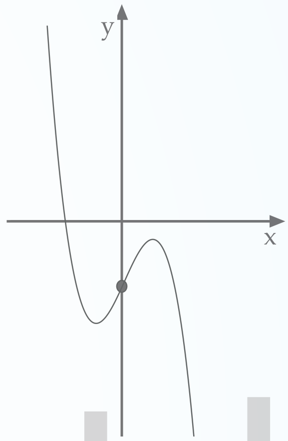
and has roots at 2 and 4.



157. Find the function whose derivative is

$$f'(x) = 1 - 3x^2$$

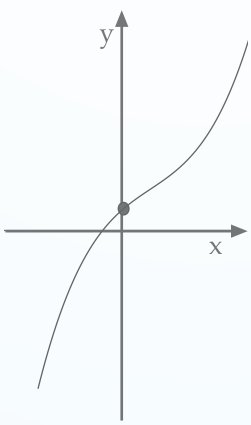
and crosses the y axis at $y = {}^{-}1$.



158. Find the function whose derivative is

$$f'(x) = 3x^2 - 2x + 1$$

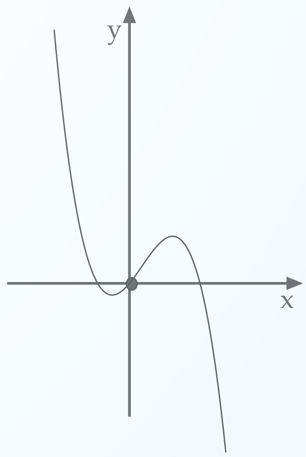
and crosses the y axis at 2.



159. Find the function whose derivative is

$$f'(x) = 1 - x^2 + x$$

and passes through the origin.



Example

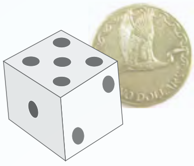
A fair die and a coin are thrown.

a) List all the possible outcomes (sample space).

Using the list of possible outcomes from a) find the probability that

b) a head occurs.

c) a head and an even number occur.



d) the number on the die is greater than or equal to three and a tail occurs.

a) Sample space

		Die					
		1	2	3	4	5	6
Coin	H	1,H	2,H	3,H	4,H	5,H	6,H
	T	1,T	2,T	3,T	4,T	5,T	6,T

b) $P(\text{head}) = \frac{6}{12} = \frac{1}{2}$ (1,H), (2,H), (3,H), (4,H), (5,H), (6,H) out of 12 possibilities.

c) $P(\text{H and even}) = \frac{3}{12} = \frac{1}{4}$ (2,H), (4,H), (6,H) out of 12 possibilities.

d) $P(\text{die} \geq 3 \text{ and T}) = \frac{4}{12} = \frac{1}{3}$ (3,T), (4, T), (5,T), (6,T) out of 12 possibilities.

Relative Frequency

Results that we obtain from repeated experiments or observations can be used to predict outcomes of future events. We express these results as a proportion of the whole called relative frequency.

For example, we may have observations on the different transport options used by students travelling to school and we use these to predict the probability of a random student using a particular method.

Alternatively, we may want to know the probability of a drawing pin landing pin up and we conduct an experiment to get some data. As we repeat an experiment a large number of times the probability approaches a consistent value. This is called **long run relative frequency**.

6. The local netball team is very good, but it has a poor 'away' record. On the basis of its record calculate the following probabilities.

	Win	Lose	Total
Home games	21	3	24
Away games	8	10	18
Total	29	13	42

a) What is the probability of it winning a home game?

b) If we know the team won a game, what is the probability it was an away game?

c) If we know the team lost a game, what is the probability it was a home game?

7. The breakdown of the sex and age of employees working at a company is given in the table below.

	Male	Female	Total
Less than 40	35	20	55
40 or over	40	65	105
Total	75	85	160

a) What is the probability of an employee selected at random being under 40 years of age?

b) Given an employee is female what is the probability they are 40 years of age or over?

c) Given an employee is under 40 years of age, what is the probability they are male?

8. An analysis at a local hospital of the cause of death of people aged between 50 and 65 and whether they smoked is given below.

	Smoking		
	Nil	Light	Heavy
Other causes	31	10	7
Cancer / heart disease	9	15	28

Find the probability that a randomly selected death is

a) of a non-smoker.

b) from cancer or heart disease.

c) from cancer or heart disease given the person smokes.

d) from cancer or heart disease given the person is a heavy smoker.

e) a heavy smoker who has died of cancer or heart disease.

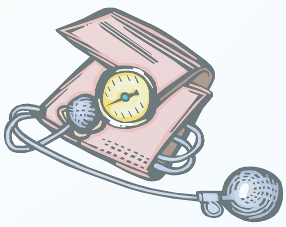
9. Free blood pressure tests conducted at a pharmacy yielded the following results.

	Male	Female	Total
Normal	46	53	99
Pre-hypertensive	40	37	77
Hypertensive	28	16	44
Total	114	106	220

a) What is the probability of a random person who was tested being hypertensive?

b) Given a person is pre-hypertensive what is the probability they are male?

c) Given a person is female what is the probability they have a normal blood pressure reading?



Risk

Risk

In statistics, risk is the probability or likelihood of an event occurring. Risk is often expressed as a decimal or percentage.

When you attempt something that could result in harm or injury you are putting yourself at risk. Risk is generally considered to have this negative connotation so in situations where this is not the case you should look to use another term to avoid confusing readers. For example, if you were looking at the data for two groups gaining employment it would be inappropriate even if mathematically correct to use the term 'risk of getting a job' as you are likely to confuse your audience. You could just use the term 'probability of getting a job'. There are many different kinds of risk: business risk, economic risk, health risk etc.

In probability we can define two types of risk: absolute risk and relative risk.

Absolute Risk

Absolute risk is the probability or percentage of subjects in any group that experience an outcome. For example, the absolute risk of a person developing lung cancer over their lifetime could be expressed as 0.11 or 11% or 1 in 9.

Relative Risk

Relative risk is used to compare the risk in two sub groups of our population. For example, we could measure the risk of people getting lung cancer as a consequence of smoking by comparing the risk of getting lung cancer for people who smoke with the risk of getting lung cancer for the group that do not smoke.

We can calculate the relative risk of getting lung cancer as a consequence of smoking (or the risk ratio) by using the formula

$$RR = \frac{\text{Risk or Prob. with condition present}}{\text{Risk or Prob. with no condition present}}$$

Where RR is the relative risk.

Consider the table below which shows that the risk of getting lung cancer among smokers is 25% while only 2% among non-smokers.

	Lung cancer	No lung cancer
Smoker	0.25	0.75
Non-smoker	0.02	0.98

To calculate the relative risk of getting lung cancer as a consequence of smoking we use the formula given above. The probability of a smoker being exposed to cancer is 0.25 while the probability of a non-smoker being exposed to cancer is 0.02.

Therefore the relative risk $= \frac{0.25}{0.02} = 12.5$.

So smokers are 12.5 times more likely than non-smokers to develop lung cancer.

Relative risk is a value that identifies how much something you do, such as maintaining a healthy weight, can change your risk compared to the risk if you're very overweight.

Values for relative risk (RR) can be any number greater than zero. If your relative risk (RR) value is 1 it means the outcome is not influenced by the factor present, i.e. the likelihood of lung cancer is not affected by smoking.

If your relative risk (RR) value is greater than 1 it means the outcome is greater when the factor is present, i.e. you are more likely to get lung cancer if you smoke.

If your relative risk (RR) is less than 1 it means the outcome is less likely when the factor is present. An example may be the relative risk of having a heart attack when taking a low dose of aspirin compared to having a heart attack and not taking any aspirin.

	Heart Attack	No heart attack
Takes aspirin	0.94%	99.06%
No medication	1.71%	98.29%

If you now calculated the relative risk of having a heart attack with or without a low dose of aspirin then

$$RR = \frac{\text{Risk of heart attack with aspirin}}{\text{Risk of heart attack with no aspirin}}$$

$$RR = \frac{0.94}{1.71}$$

$$RR = 0.55$$

A relative risk less than 1 is correct but difficult to interpret so often the problem is restated so that the increased risk is the numerator and the relative risk is then greater than 1.

In this case we would state that the relative risk of having a heart attack by taking no medication compared to taking a low dose of aspirin is

$$RR = \frac{\text{Risk of heart attack with no aspirin}}{\text{Risk of heart attack with aspirin}}$$

$$RR = \frac{1.71}{0.94}$$

$$RR = 1.82$$

The risk of a heart attack by taking no medication is 1.8 times the risk compared to taking a low dose of aspirin.

Example (Using graphics calculator)

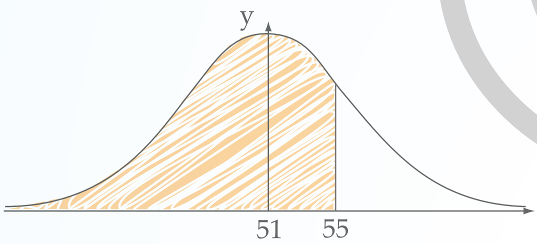
The mean mark in a school's Year 12 mathematics exam was 51% with a standard deviation of 16%. Assuming the marks are normally distributed find the probability that a student selected at random scored

a) less than 55%.

b) between 35% and 45%. Hence find how many out of 250 students you expect to get between 35% and 45%.

Using your TI-84 Plus

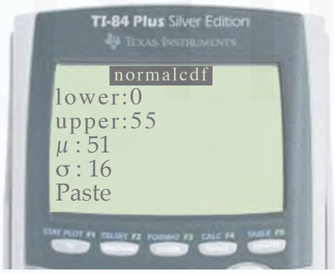
Even when using your calculator sketch at least one normal curve so you can visualise the area you require. We can see we expect an answer over 0.5.



Access the distribution menu and either scroll down or select 2 to get normalcdf(.

[2nd] [DISTR] [2]

a) **With P(X < 55) and μ = 51, σ = 16, enter lower : 0, upper : 55, μ : 51 and σ : 16.**



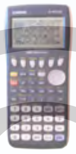
This gives 0.5979886673 or 0.5980 (4 dp).

b) **The sketch for part b) is on the right. For P(35 < X < 45) and μ = 51, σ = 16. Enter lower : 35, upper : 45, μ : 51 and σ : 16.**

This gives 0.1952 (4 dp).

Number of students = 0.1952 x 250

= 48 (rounding down)



Using your Casio 9750GII

The sketch for part a) is on the left. It is important that you sketch at least one normal curve so you can visualize the area you require.

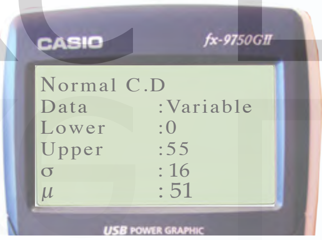
From the Main menu select STAT then DIST, NORM and Ncd.

STAT	DIST	NORM	Ncd
[2]	[F5]	[F1]	[F2]

For the Casio use the format Lower, Upper, σ and μ.

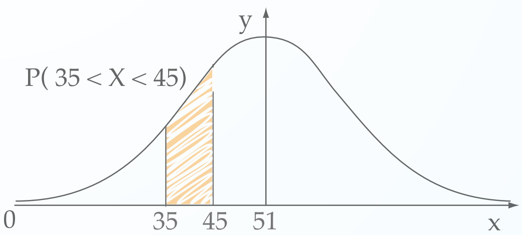
a) **With P(X < 55) with σ = 16 and μ = 51. enter Lower : 0, Upper : 1.5, σ : 16 and μ : 51.**

[0] [EXE] [5] [5] [EXE] [1] [6]
[EXE] [5] [1] [EXE] [EXE]



getting the probability = 0.5980 (4 dp).

b) **For P(35 < X < 45)**



Enter Lower : 35, Upper : 45, σ : 16 and μ : 51.

[3] [5] [EXE] [4] [5] [EXE]
[1] [6] [EXE] [5] [1] [EXE] [EXE]

This gives 0.1952 (4 dp).

Number of students = 0.1952 x 250

= 48 (rounding down)

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418. a) $5000 = 50\,(1.095)^T$

$100 = (1.095)^T$

$T = \dfrac{\log 100}{\log 1.095}$

T = 50.7 years

T = 51 years round up

i.e. in 2031.

b) $500\,(1.095)^N = 25 \times 68\,(1.030)^N$

N = 20 years rounding up

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419. a) k = 43.527(8)

$(d + 0.5)^{1.2} = 8.7055$

d = 5.6 km

b) $48 = 120 \times 1.0151^{(13 - t)}$

t = 74.1 or 74 weeks

c) $t_1 + t_2 = 27.10$

$t_1 = 0.45c$

$t_2 = 5 + 0.015c^2$

$0.45c + 5 + 0.015c^2 = 27.10$

$0.015c^2 + 0.45c - 22.10 = 0$

$c = {}^{-}56.21,\ 26.2$

Answer 26 characters

Pages 59 – 64

Practice External Assessment Task

Question One

a) $2x^3 - 5x^2 - 13x + 30$ **A**

b) $\log\left(\dfrac{A^3B}{\sqrt{C}}\right)$ **A**

c) i) $x = \dfrac{1}{3}, 1$ **A**

ii) Discriminant

$0 = k^2 - 4 \times 3 \times 4$

$k = \pm 4\sqrt{3}$ (±6.928) **E**

d) $x + 3x + (6x - 12) = 128$

Mary is 72 years **M**

A part eqn. or M eqn. & soln.

e) i) Factorise numerator and denominator

$(x-4)(x+1) = 3(x-4)$

$x = 4$ **M**

A some simplification or M complete simplification and solution.

ii) Both x = 2 and $x = {}^{-}2$ make the original equation undefined. **E**

Question Two

a) A = 42.89 **A**

k = 1.118

t = 13.7 years **M**

Sept 2021 or Jan 2022 **E**

Depends upon whether t rounded or not as to the month it reaches 200.

b) i) x = 1.672 (4 sf) **A**

ii) $x = 5, {}^{-}2$ **A**

Excludes $x = {}^{-}2$ **M**

as all log expressions must be positive. **E**

c) $2n - 14 > 9n$

${}^{-}7n > 14$

$n < {}^{-}2$ **A**

d) $p(x+2)^2 + q(x+2) + r$

$= p(x^2 + 4x + 4) + qx + 2q + r$

$= px^2 + (4p + q)x + 4p + 2q + r$

Equating to $2x^2 - 7x - 4$

$p = 2,\ q = {}^{-}15,\ r = 18$ **M**

Question Three

a) i) $(18 - 2x)(15 - x) = 162.5$

$2x^2 - 48x + 107.5 = 0$

$4x^2 - 96x + 215 = 0$ **A**

ii) $(2x - 43)(2x - 5)$ **A**

x = 2.5, 21.5

Ignore 21.5

Length = 13 m

Width = 12.5 m **M**

b) $4x = (x+1)^2$

$x^2 - 2x + 1 = 0$

$x = 1$ **M**

c) $x^2 + ({}^{-}x - k)^2 = 8$

$2x^2 + 2kx + k^2 - 8 = 0$

Discriminant > 0

$4k^2 - 4 \times 2(k^2 - 8) > 0$

${}^{-}4k^2 + 64 > 0$

${}^{-}4 \le k \le 4$ **E**

A correct simplified quadratic or M discriminant > 0.

d) $T_1 = a$

$T_n = a + (n-1)d$

$S_n = n \times \left(\dfrac{\text{first} + \text{last}}{2}\right)$

$S_n = n\left(\dfrac{a + (a + (n-1)d)}{2}\right)$

$S_n = \dfrac{n}{2}(a + a + (n-1)d)$

$S_n = \dfrac{n}{2}(2a + (n-1)d)$ **E**

Question Three cont...

e) 25 litres a month at a cost of $26 **A**

Sufficiency

For Question 1 students require two of A for Achievement or two of M for Merit or one or two of E for Excellence.

For Question 2 students require two of A for Achievement or two of M for Merit or one or two of E for Excellence.

For Question 3 students require two of A for Achievement or two of M for Merit or one or two of E for Excellence.

Overall students require

Two or more Achievement questions or better for overall Achievement.

Two or more Merit questions plus one Achievement question or better for overall Merit.

Two or more Excellence questions plus one Achievement question or better for overall Excellence.

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach, but the results should be broadly similar.